Notch Sensitivity of Ceramic Composites with Rising Fracture Resistance

Michael A. Mattoni* and Frank W. Zok*

Materials Department, University of California, Santa Barbara, California 93106

A theoretical framework is developed for the notched strength of ceramic composites that exhibit rising fracture resistance. It is based on established concepts of crack stability under stress-controlled loadings. On using a linear representation of the resistance curve (expressed in terms of an energy release rate), straightforward analytical solutions are obtained for the strength as well the amount of stable crack growth preceding fracture and the associated fracture resistance. Calculations are performed for several test configurations commonly used for material characterization, including single- and double-edge-notched tension, center-notched tension, and single-edge-notched bending. The results reveal salient trends in strength with notch length and specimen geometry. An assessment of the theory is made through comparison with experimental measurements on an all-oxide fiber composite. Transitions in the degree of notch sensitivity with notch length are identified and explored. The utility of the theoretical results both for rationalizing the trends in measured notch strength and for guiding experimental studies of notch sensitivity is demonstrated.

I. Introduction

Crack growth in continuous fiber ceramic composites (CFCCs) is characterized by a rising fracture resistance beyond the initiation toughness, \( G_{\text{init}} \), a manifestation of fiber bridging in the crack wake. When the bridging fibers furthest from the crack tip disengage, the resistance reaches a steady-state value, \( G_{\text{ss}} \), independent of further crack growth. The steady-state bridging toughness is \( \Delta G_{\text{ss}} = \beta \sigma u_\alpha \), where \( u_\alpha \) is the critical crack opening displacement to disengage the bridging fibers, \( \sigma \) is a measure of the pullout stress, and \( \beta \) is a nondimensional parameter of order unity.

The amount of crack growth \( \Delta a \) needed to achieve steady state is dictated by a characteristic bridging length scale: \( \Delta a = \frac{E_{\text{eff}}}{\sigma_\alpha} \), where \( E_{\text{eff}} \) is the plane strain Young’s modulus. For typical values of the material parameters (\( E = 100 \) GPa, \( \sigma_\alpha = 30 – 100 \) MPa), the bridging toughness is \( 10^4 – 10^3 \) J/m² and the characteristic length scale is \( 10 – 100 \) mm. The implications are 2-fold. (i) Significant toughening can be obtained at steady state, relative to the initiation toughness. (ii) Because of the large amount of crack growth needed to achieve steady state, the benefits of the rising resistance curve in composite strength are not fully realized. That is, crack growth becomes unstable with respect to the applied stress long before steady state is achieved.

The objective of this paper is to address the related issues of crack stability and notch sensitivity of strength in ceramic composites that exhibit a rising fracture resistance, both theoretically and experimentally. On the theoretical front, an analytical framework is developed for describing crack stability in an orthotropic material. It is based on comparisons of the energy release rates of common notched test configurations with the material’s fracture resistance. Analogous treatments of notch sensitivity have been used previously for other materials that exhibit rising fracture resistance, including metal-, whisker- and fiber-reinforced ceramics and ceramics toughened by grain pullout. For our purpose, the resistance curve \( G_R(\Delta a) \) is assumed to be linear over the range of crack growth preceding instability. Furthermore, the size of the inelastic deformation zone at the crack tip is considered to be small in relation to the geometric and the bridging length scales, i.e., small-scale yielding conditions are obtained. Effects of inelasticity are thus embodied in the initiation toughness. Elsewhere the material is assumed to be elastically orthotropic. To assess the theory, comparisons are made with experimental measurements of fracture-resistance curves and notch sensitivity of strength in a porous-matrix all-oxide CFCC.

It is emphasized that the theoretical approach is based on an assumed form of the resistance curve, notably linear; no attempt is made to model the resistance curve itself. The latter problem falls in the domain of bridging micromechanics, a field that has been explored exhaustively over the past decade. Although micromechanics provides a more fundamental link between the bridging phenomenon and the macroscopic response, it requires numerical solution of the integral equations governing the crack-opening displacement profile, the crack-tip stress intensity factor, and the bridging traction law. As a consequence, it is likely to find limited use in the engineering community. Arguably, for the purpose of predicting the strength of notched or flawed structures, micromechanics is not essential. Instead, only a convenient functional form for the resistance curve is required. This phenomenological approach has proved to be useful in addressing strength variability in ceramics. Furthermore, as illustrated in the present paper, in cases where the resistance curve is linear (as it is in the oxide composite of interest), straightforward analytical solutions for notched strength are obtained. These solutions allow probing of the effects of specimen type and size on notch sensitivity and identifying transitions in behavior.

II. Theory of Crack Stability

To begin, it is instructive to consider the case of a specimen of width \( W \), containing a small center notch of length \( 2a_0 \), and loaded in tension perpendicular to the notch plane along one of the principal material axes. In the limit where \( a_0 \ll W \), the energy release rate is given by

\[
G = g(\rho,\lambda) \left( \frac{\sigma^2 \pi a}{E} \right)
\]

where \( \sigma \) is the applied tensile stress, \( g(\rho,\lambda) \) characterizes the effect of orthotropy, and \( \rho \) and \( \lambda \) are defined by

\[
\lambda = \frac{E_2}{E_1}
\]

\[
\rho = \frac{E_1}{E_1 + E_2}
\]
Griffith equation; no strength benefit is derived from the increase; the strength is controlled by the initiation toughness via the growth. Consequently, interestingly, since both (from Eq. (5)) to be characterized by the tearing modulus: $T$ is $G_0/\mu$, the stress at which unstable crack growth occurs unstably under a decreasing stress. Consequently, the strength is controlled by the initiation toughness via the Griffith equation; no strength benefit is derived from the increasing resistance. Conversely, if $dG/da > T$ at $G = G_0$, continued crack growth requires an increase in stress beyond that for initiation. The transition from unstable to stable growth occurs at a critical tearing modulus, $T_r$, at which $dG/da = T$ at $G = G_0$. From Eq. (5) or inspection of Fig. 1, the critical tearing modulus in nondimensional form is

$$\frac{T_r W}{G_0} = \frac{W}{a_0} + \left( \frac{1}{H(a_0/W)} \right) \frac{dH(a_0/W)}{d(a_0/W)}$$

Interestingly, since both $T_r$ and $G_0$ scale with $g(\rho, \lambda)$, the nondimensional result in Eq. (6) is independent of material orthotropy.

The same approach is used to address the stability of cracks in finite bodies. The effects of $a/W$ on the energy release rate can be described generically by

$$G = H(a/W) g(\rho, \lambda) \left( \frac{\sigma^2 \pi a}{E} \right)$$

where $H(a/W)$ is a nondimensional function that depends on both the specimen geometry and the loading configuration. Combining Eqs. (5) and (7), the critical tearing modulus becomes

$$\frac{T_r W}{G_0} = \frac{W}{a_0} + \left( \frac{1}{H(a_0/W)} \right) \frac{dH(a_0/W)}{d(a_0/W)}$$

In the limit where $a_0/W \rightarrow 0$, the first term on the right side dominates and the critical tearing modulus reduces to the 1 in Eq. (6), as required.

If $T \leq T_r$, the instability occurs at the initiation toughness at the Griffith stress, $\sigma_0$, written in nondimensional form as

$$\frac{\sigma^2 \pi W}{E G_0} = \frac{1}{(a_0/W) H(a_0/W) g(\rho, \lambda)}$$

Conversely, if $T > T_r$, stable crack growth occurs until the conditions in Eq. (5) are satisfied. This scenario is represented by the tangent construction in Fig. 1(b). Provided that instability occurs within the linear region of the resistance curve, the critical values of crack length, $a_0$, resistance, $G_0$, and stress, $\sigma_0$, are found (from Eq. (5)) to be

$$a_0 = \frac{a_0}{W} - \frac{1}{\sigma_0} \left( \frac{1}{H(a_0/W)} \right) \frac{dH(a_0/W)}{d(a_0/W)} + \frac{G_0}{T W}$$

Fig. 1. Tangent construction plots, showing the following: (a) the transition from stable to unstable crack growth for $a_0/W = 0$; (b) stable crack growth (starting at $\sigma_0$) and onset of instability (at $\sigma_0$) in the linear portion of the resistance curve for finite $a_0/W$; and (c) unstable crack growth at the steady-state toughness. (Here the material is assumed to be elastically isotropic such that $g(\rho, \lambda) = 1$.)
where

\begin{equation}
G_i = \frac{WT}{G_0} \left( \frac{W}{a_i} + \left( \frac{1}{H(a/W)} \right) \frac{dH(a/W)}{d(a/W)} \right)^{-1} \tag{10b}
\end{equation}

\begin{equation}
\sigma_i^2 \pi W = \frac{TW}{EG_0} \left( \frac{1}{(a/W) H(a/W) g(p, \lambda)} \right) \times \left( \frac{W}{a_i} + \left( \frac{1}{H(a/W)} \right) \frac{dH(a/W)}{d(a/W)} \right)^{-1} \tag{10c}
\end{equation}

If the predicted critical resistance \( G_i \) exceeds \( G_{cr} \) (Fig. 1(c)), the strength is obtained at \( G = G_{cr} \) at a total crack length,

\[ a_{cr} = a_0 + \Delta a_{cr} = a_0 + G_{cr} - G_0 \tag{11} \]

The corresponding stress is

\[ \sigma_i^2 \pi W = \left\{ H(a/W) g(p, \lambda) \left[ \frac{a_0}{W} \left( \frac{G_0}{G_{cr}} \right) + \frac{G_0}{TW} \left( 1 - \frac{G_0}{G_{cr}} \right) \right] \right\}^{-1} \tag{12} \]

### III. Results Common Test Configurations

Results for a number of notched configurations commonly used for material characterization have been computed using the analysis in Section II along with established solutions for \( H(a/W) \). The configurations of interest are as follows: (i) center-notched tension (CNT), (ii) single-edge-notched tension (SENT), (iii) double-edge-notched tension (DENT), (iv) single-edge-notched (pure) bending (SENB), and (v) single-edge-notched beam in three-point bending (SENTPB) with a span to width ratio of \( S/W = 4 \). Pertinent solutions for \( H(a/W) \) are summarized in Table I; \(^1^0\) each is accurate within 0.5% over the entire range of \( a/W \).

Straightforward analytical results for the critical tearing modulus are obtained for the limiting case in which the uncracked ligament becomes small; i.e., \( a_0/W \rightarrow 1 \). For SENT, SENTPB, and SENB, the limiting solutions for \( H(a/W) \) take the form

\[ H(a/W) = \frac{\beta_1}{(1 - a_0/W)^3} \tag{13} \]

where \( \beta_1 \) depends on the loading configuration but is independent of \( a_0/W \). Combining this result with Eq. (8) yields

\[ T,W = 2 + Wa_0 \]

\[ \frac{G_0}{1 - a_0/W} \]

Similarly, for both CNT and DENT, the limiting solutions for \( H(a/W) \) take the form

\[ H(a/W) = \frac{\beta_2}{(1 - a_0/W)} \tag{15} \]

where \( \beta_2 \) is another constant, independent of \( a_0/W \). Combining with Eq. (8) yields

\[ \frac{T,W}{G_0} = \frac{1}{(a_0/W)(1 - a_0/W)} \tag{16} \]

Figure 2 shows the variation in the critical tearing modulus, \( T,W/G_0 \), with notch length for the five loading configurations \(^1\) (Fig. 1) as well as for the limiting cases of short notches and short ligaments. In each of the former cases, \( T,W/G_0 \) exhibits a minimum at a notch length in the range \( a_0/W \approx 0.3–0.5 \) and becomes large in the limits \( a_0/W \rightarrow 0 \) and \( a_0/W \rightarrow 1 \). Thus, for a prescribed value of \( T,W/G_0 \), crack stability is only obtained over an intermediate range of \( a_0/W \); both short notches and long notches promote unstable growth. Furthermore, \( T,W/G_0 \) approaches the short notch limit (Eq. (6)) when \( a_0/W \leq 0.1 \) and the pertinent short ligament limit (Eq. (14) or (16)) when \( a_0/W \approx 0.9 \).

The figure also shows the sensitivity of \( T,W/G_0 \) to the specimen type. DENT exhibits the lowest value over the entire range of \( a_0/W \) and hence the greatest propensity for stable crack growth. SENT is at the other extreme with the highest value of \( T,W/G_0 \), by as much as a factor of 2–3 relative to that for DENT. CNT, SENTPB, and SENB give intermediate results.

The effects of the normalized tearing modulus, \( T,W/G_0 \), on crack stability in the SENB specimen are illustrated in Fig. 3. For the purpose of identifying the range of notch lengths over which crack stability is obtained at initiation, the critical tearing modulus is replotted in Fig. 3(a) and compared with selected values of \( T,W/G_0 \) for SENTPB, 10, 20, and 30. The critical points are indicated by the solid symbols and labeled a–f. The amount of crack growth (Eq. (10a)),

\[ \sqrt{\pi}(1 + 2a/W)(1 - a/W)^{3/2} \]

### Table I. Summary of the Functions \( H(a/W) \) for Common Loading Configurations

<table>
<thead>
<tr>
<th>Loading configuration</th>
<th>( [H(a/W)]^{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNT</td>
<td>( [1 - 0.025(a/W)^2 + 0.06(a/W)^4] ) ( \frac{\sec \left( \frac{\pi a}{2W} \right)}{\sqrt{1 - a/W}} )</td>
</tr>
<tr>
<td>SENT</td>
<td>( \frac{2W}{\pi a} \tan \left( \frac{\pi a}{2W} \right) \left( 0.752 + 2.02(a/W) + 0.37 (1 - \sin \left( \frac{\pi a}{2W} \right))^2 \right) \cos \left( \frac{\pi a}{2W} \right) )</td>
</tr>
<tr>
<td>DENT</td>
<td>( \frac{1.122 - 0.561(a/W) - 0.205(a/W)^2 + 0.471(a/W)^3 - 0.190(a/W)^4}{\sqrt{1 - a/W}} )</td>
</tr>
<tr>
<td>SENB</td>
<td>( \frac{\frac{2W}{\pi a} \tan \left( \frac{\pi a}{2W} \right) \left( 0.923 + 0.199 (1 - \sin \left( \frac{\pi a}{2W} \right))^4 \right) \cos \left( \frac{\pi a}{2W} \right)}{\sqrt{1 + 2a/W}(1 - a/W)^{3/2}} )</td>
</tr>
<tr>
<td>SENTPB (( S/W = 4 ))</td>
<td>( \frac{1.99 - (a/W)(1 - a/W)(2.15 - 3.93(a/W) + 2.7(a/W)^2)}{\sqrt{\pi}(1 + 2a/W)(1 - a/W)^{3/2}} )</td>
</tr>
</tbody>
</table>

\(^1\)Results for SENTPB and SENB are almost indistinguishable from one another. For clarity, the former results are not included in the figure.
the fracture resistance (Eq. (10b)), and the stress at instability (Eq. (10c)) are plotted in Figs. 3(b–d), respectively, along with the critical points, a–f. Outside the range that leads to stable crack growth (e.g., to the left of b or to the right of e for $TW/G_0 = 20$), unstable growth occurs at $G_f/G_0 = 1$ at the Griffith stress (Eq. (9), Fig. 3(c)). In the intermediate range of $a_o/W$, some stable crack growth is obtained (Fig. 3(b)) and the strength is elevated relative to the Griffith stress. The amount of strength elevation and the range of notch lengths over which this benefit is realized increases with increasing $TW/G_0$.

It is noteworthy that when viewed over an intermediate range of $a_o/W$, the shapes of the strength curves (Fig. 3(d)) are not strikingly different from one another. Consequently, experimental measurements of strength for several notch sizes could be misinterpreted using linear elastic fracture mechanics (LEFM), assuming a uniquely valued toughness, and thus the effects of the resistance curve could be overlooked. A more critical distinction can be made by also performing tests on specimens of different widths to vary $TW/G_0$, or on other specimen types, especially ones that yield substantially different values of $T_cW/G_0$.

To illustrate the effects of specimen type, results are presented in Fig. 4(a) for the two extreme cases: SENT and DENT. To distinguish the results from those obtained from LEFM, they are replotted in Fig. 4(b) in terms of $\sigma_f/\sigma_0$. The latter ratio represents the strength elevation above the Griffith stress due to the resistance curve. The results demonstrate that the beneficial effects of the rising fracture resistance are more pronounced in DENT than in SENT. For instance, for $TW/G_0 = 20$, the maximum strength elevation above the initiation stress is only about 18% at $a_o/W = $.

---

**Fig. 2.** Effects of notch length and specimen type on the critical tearing modulus above which stable crack growth occurs.

**Fig. 3.** Effects of tearing modulus on crack stability and strength for SENB.
IV. Experimental Procedure

The CFCCs used for experimental measurements comprise seven layers of 3000 denier Nextel 720 fiber cloth in an 8-harness satin weave and a porous mullite–alumina matrix. The matrix was introduced in two steps: (i) pressure-assisted infiltration of a mullite–alumina slurry through the fiber preform and (ii) impregnation and pyrolysis of an Al₂Cl(OH)₅ solution (an alumina precursor). Processing details were essentially identical to those described earlier with two exceptions. First, the concentration of the precursor solution was increased to give a volumetric yield of 6.4%, compared with 3% in previous studies. The more concentrated solution allows a prescribed matrix density to be achieved with fewer impregnation and pyrolysis cycles. In the present study, four such cycles were used, yielding a final matrix porosity of 32%. Second, to prevent segregation of the precursor during drying, the panels were gelled immediately after precursor impregnation by immersing them in an NH₄OH environment for 6 h. The absence of segregation was confirmed through a series of matrix hardness measurements across the plate thickness. The composite panel thickness was 3.3 mm.

As a parenthetical note, the matrix in the present composite had been designed to be considerably harder than that in composites described in earlier studies. This selection was motivated by the attendant improvements in the off-axis properties, e.g., 0°/90° shear strength. Although not reported here, significant elevations in shear properties are indeed obtained with increasing matrix hardness. A comparative study on the effects of matrix hardness on the composite performance in both matrix- and fiber-dominated loadings will be reported elsewhere.

In-plane elastic properties were obtained from unnotched tension and Iosipescu shear tests, performed parallel to the fiber directions. The tensile specimens were of a dog-bone configuration with a gauge section 8 mm wide and 60 mm long. The Iosipescu specimens were 19 mm wide and contained a pair of V-notches with an included angle of 105°, to a depth of 3.8 mm. Both specimen types were instrumented with strain gauges. The key results from these tests are as follows: E₁ = 64 GPa, ν₁₂ = 0.13, and μ₁₂ = 14 GPa. Combining with Eqs. (3) and (4) yields p = 2.15 and g(p) = 1.51.

Additional mechanical tests were performed in the 0°/90° orientation on three types of notched specimens: SENTPB, SENT, and DENT. The geometric parameters were selected to probe the effects of W at fixed a₀/W and the effects of a₀/W at fixed W. The pertinent values are listed in Table II. For SENTPB, the span length was S/W = 4. The length of the SENT specimens was H/W = 1.0; for the DENT specimens, H/W = 3. Crack-growth measurements were made using crack gauges affixed to the specimen surface (Krack-Gauge, model AOS-CE, Hartrun Corp., St. Augustine, FL). These measurements were checked against direct optical measurements on the opposing surface. Image contrast was enhanced by coating the viewing surfaces with Au–Pd. The cracks were observed to be essentially planar, with no evidence of inelastic deformation or damage away from the crack tip. The nominal fracture resistance was calculated via Eq. (7), using the
DENT

†

ness. The transition between the two corresponds to Griffith stress (Eq. (9)), calculated using the initiation tough-

tension. The results are plotted in Fig. 6. Also shown for comparison are the values obtained from LEFM: notably, the

nonlinearity beyond this point, a manifestation of large-scale bridging. In all pertinent cases, the peak stress was attained when the crack was within the linear portion of the

resistance curve.

The tearing modulus was obtained through a linear regression analysis of the

measured load and crack length, coupled with the computed value of \( R(a/W) \) and \( g(p) = 1.51 \). For SENT, the effects of specimen rotation in the gripped sections were incorporated using the approach described in the Appendix. The SENTPB specimens were also instrumented with a spring-loaded one-armed extensometer to monitor load–line displacement. The work of fracture, obtained from the area under the load–displacement curve, was used as a measure of the steady-state toughness.\(^{14}\) Base-line values of unnotched strength were obtained from uniaxial tension tests (described above) and four-point bend tests. The bend specimens were either 6 or 12 mm wide and loaded with a span \( S/W = 9 \).

The experimental results are summarized in Figs. 6 and 7. In all cases, stable crack growth had been observed before the stress maximum and the resistance curves were essentially linear. All the tensile geometries yielded common resistance parameters:\(^{1} \) notably, \( G_0 = 900 \pm 200 \text{ J/m}^2 \) and \( T = 400 \pm 100 \text{ kJ/m}^3 \). In contrast, the SENTPB specimens exhibited significantly higher fracture resistance, characterized by \( G_0 = 1500 \pm 150 \text{ J/m}^2 \) and a tearing modulus \( T = 950 \pm 100 \text{ kJ/m}^3 \). It is surmised that the differences are associated with the \( T \)-stresses and their effect on inelasticity in the notch tip region. That is, for SENTPB, the \( T \)-stress is positive over the range of \( a_0/W \) used in the present experiments.\(^{9} \) When positive, the \( T \)-stress promotes matrix microcracking ahead of the notch tip which, in turn, reduces the normal stress along the incipient fracture plane and hence increases fracture resistance.\(^{13} \) In contrast, for the tensile geometries, the \( T \)-stress is negative over the pertinent range of \( a_0/W \), suppressing the inelasticity that leads to increased resistance.

The one notable exception to the behavior described above pertains to the narrowest SENTPB specimens. In this case, the fracture resistance was anomalously low (relative to that measured on the wider specimens) and the notched strength was essentially equal to the unnotched strength. Evidently, fracture is controlled by the net-section stress rather than the \( K \)-field associated with the notch. Consequently, the analysis presented here does not apply.

V. Comparisons between Experiment and Theory

Notched strength predictions were made using the analysis presented in Section II and the range of resistance parameters shown in Fig. 7: \( G_0 = 1500 \pm 150 \text{ J/m}^2 \) and \( T = 950 \text{ kJ/m}^3 \) for flexure and \( G_0 = 900 \pm 200 \text{ J/m}^2 \) and \( T = 400 \text{ kJ/m}^3 \) for tension. The results are plotted in Fig. 6. Also shown for comparison are the values obtained from LEFM: notably, the Griffith stress (Eq. (9)), calculated using the initiation toughness. The transition between the two corresponds to \( T_s/W/G_0 \). Broadly, the predictions correlate well with the experimental measurements.

Viewed collectively, the results provide insights into the transitions in behavior with increasing notch length. For instance, for the SENT specimens with \( a_0/W = 0.5 \), three domains exist: (i) net-section-stress-dominated, denoted I, for \( a_0 \leq 1 \text{ mm} \), (ii)
and the steady-state toughness ($G_{ss}$) results are presented only for the transition III row of circles), the critical notch length is given by Eq. (8); for the open circles show the behavioral transitions. For the transition II a SENTPB geometry with $T$-dominated, denoted III, for $J/m^2$ and $1500 J/m^2$, well beyond the values relevant to the regime where instability occurs.

![Fracture resistance curves](image)

**Fig. 7.** Typical fracture resistance curves obtained from SENTPB and SENT tests. Lines represent envelope of fit values, with $G_0 = 1500 \pm 150 J/m^2$ and $T = 950 kJ/m^3$ for flexure and $G_0 = 900 \pm 200$ and $T = 400$ for tension. The steady-state toughness obtained from SENTPB tests is $G_{ss} = 5300 \pm 400 J/m^2$, well beyond the values relevant to the regime where instability occurs.

initiation-dominated, denoted II, for $1 mm \leq a_0 \leq 5 mm$, and (iii) $T$-dominated, denoted III, for $a_0 = 5 mm$. For the SENTPB specimens, the transition from I to II occurs at stresses that are essentially the same as the unnotched strength. The implication is that the behavior transitions directly from I to III without passing through II.

To further explore these transitions, calculations have been performed for other values of $a_0/W$. Representative results for the SENTPB geometry with $a_0/W$ in the range 0.25–0.75 are plotted in Fig. 8; similar trends are obtained for the other geometries. For clarity, results are presented only for the average values of the initiation toughness ($G_0 = 1500 J/m^2$), the tearing modulus ($T = 950 kJ/m^3$), and the steady-state toughness ($G_{ss} = 5300 J/m^2$). Furthermore, they are presented over a broad range of notch lengths to capture the full range of behavior. Two new features are notable. First, there exists an additional domain, denoted IV, for the longest notches, where the strength is dictated by the steady-state toughness. In this domain, the notched strength asymptotically approaches an inverse square root dependence on notch length, analogous to that of the initiation-dominated domain. Second, the notch lengths corresponding to the transitions increase substantially with increasing $a_0/W$. For instance, for a very deep notch, characterized by $a_0/W = 0.75$, the transition from I to II occurs for $a_0 = 14 mm$. In contrast, with notches of this length in a specimen with $a_0/W = 0.25$, the behavior is well within domain IV. As a consequence, striking differences in behavior would be obtained over the practical range of notch lengths, from 2 to 14 mm. In the former case ($a_0/W = 0.75$), the strength would be indistinguishable from the unnotched strength; in the latter ($a_0/W = 0.25$), significant notch sensitivity would be apparent. The latter tests are recommended to most critically probe notch effects.

![Effects of $a_0/W$ on notch strength of SENTPB specimen](image)

**Fig. 8.** Effects of $a_0/W$ on notched strength of SENTPB specimen. The open circles show the behavioral transitions. For the transition II $\rightarrow$ III (top row of circles), the critical notch length is given by Eq. (8); for the transition III $\rightarrow$ IV (lower row of circles), it is given by Eq. (11).

### VI. Concluding Remarks

A theoretical framework for predicting the notch sensitivity of strength of ceramic composites with rising fracture resistance has been presented. An attractive feature of the resulting solutions is their relatively simple analytical form: a consequence of the assumed linear form of the resistance curve. The framework provides a consistent description of the strength of the oxide composite studied here when the notch length is large and the corresponding strength is low in relation to the unnotched strength. Parametric studies illustrate the salient features in the transitions in fracture behavior and their dependence on both the notch length and its normalized value ($a_0/W$).

In addition to its use in rationalizing experimental results, the theoretical framework should find utility in designing critical experiments to probe notch sensitivity of other composite materials. For instance, with preliminary estimates of $G_0$ and $T$, calculations could be performed to identify the specimen sizes and types for which stable crack growth should occur and experiments performed accordingly. This combined theoretical/experimental approach is expected to yield significantly improved understanding of notch sensitivity relative to that obtained through an ad hoc experimental program.

On a cautionary note, the framework is not expected to apply to composites that exhibit large-scale inelasticity in the notch-tip region. In composites with weak (porous) matrices, such inelasticity can occur through a shear mode parallel to the loading direction. Alternatively, in systems with strong (dense) matrices, it can occur in a tensile mode through multiple matrix cracking. When the spatial extent of the inelasticity becomes comparable to the structural dimensions, the inelasticity must to be taken into account explicitly and not simply embodied in the initiation toughness.

Finally, the degree of notch sensitivity displayed by the present oxide composite is significantly greater than that in systems with weaker matrices. However, the lower notch sensitivity associated with weaker matrices is accompanied by reductions in off-axis properties, notably the $0^\circ/90^\circ$ shear strength. Trade-offs between fiber-dominated and matrix-dominated properties are the subject of a current investigation.

### Appendix

Experience with the SENT geometry indicates that small amounts of specimen rotation can occur under nominally fixed grip conditions, thereby altering the associated stress intensity factor. The magnitude of this effect is determined from measurements of the crack-mouth-opening displacement (CMOD) coupled with the following analysis.

The stresses at the specimen ends can be partitioned into two components: (i) a uniform tensile stress, $\sigma_t$, equal to the average measured stress and (ii) a bending stress, $\sigma_{b,\text{p}}$, proportional to $\sigma_t$. 


but of unknown magnitude. The stress intensity factor associated with these two stress components is

\[
K = F_t(a/W) Y(p) \sigma_Y \sqrt{a} - F_b(a/W) Y(p) \sigma_b \sqrt{a} \\
= F_t(a/W) Y(p) \sigma_Y \sqrt{a} 
\]

where

\[
Y(p) = 1 + 0.1(\rho - 1) - 0.016(\rho - 1)^2 + 0.002(\rho - 1)^3 
\]

(A-1)

Here, \(F_t(a/W)\) and \(F_b(a/W)\) are the nondimensional functions for SENT and SENB given in handbooks,\(^{10}\) \(F_t(a/W)\) is the function that characterizes the stress intensity factor in the present test configuration, and \(Y(p)\) accounts for material orthotropy.\(^7\) The corresponding CMOD is

\[
\delta = g(p, \lambda) \left[ V_t(a/W) \left( \frac{4\sigma_Y a}{E} \right) - V_b(a/W) \left( \frac{4\sigma_b a}{E} \right) \right] 
\]

(A-2)

where \(V_t(a/W)\) and \(V_b(a/W)\) are the nondimensional functions for CMOD obtained from handbooks, and \(g(p, \lambda)\) characterizes the material orthotropy as described in Section II. Rearranging Eq. A-3 yields the stress ratio:

\[
\frac{\sigma_b}{\sigma_Y} = \frac{V_t}{V_b} - \frac{Y(p)}{g(p, \lambda)} \left( \frac{\delta E}{4\sigma_Y a} \right) 
\]

(A-3)

Combining Eqs. A-1 and A-4 gives the normalized stress intensity factor as

\[
F_b = \frac{K}{\sigma_Y \sqrt{a}} \\
= F_t - F_b \left( \frac{\sigma_Y}{\sigma_b} \right) \\
= F_t - \frac{F_b}{V_b} \left[ V_t + \frac{Y(p)}{g(p, \lambda)} \left( \frac{\delta E}{4\sigma_Y a} \right) \right] 
\]

(A-4)

The stress intensity factor had been obtained from Eq. A-5 coupled with measurements of \(\delta\) in the elastic regime and the known values of the other parameters.

The degree to which the bending stress had been reduced relative to the uniform displacement condition was ascertained from a comparison of the experimentally determined stress ratio in Eq. A-4 with the predicted value. The results are plotted in Figure 9. Evidently the experimental results lie close to, but slightly below, those for the uniform displacement condition.\(^{18}\) The dashed line in the figure shows an empirical fit through the data, given by the product of the uniform displacement solution and \(m(a/w) = 1 - 0.5(a/w)(1 - a/W)\). This fit accurately represents the experimental data and provides the correct limits to the stress ratio in the limits of \(a/W \to 0\) and \(a/W \to 1\).

**Nomenclature**

- \(a\) Crack length
- \(a_0\) Initial crack or notch length
- \(a_c\) Total crack length at unstable crack growth
- \(a_n\) Total crack length at onset of steady state
- \(a_n^\infty\) Characteristic bridging length scale
- \(E\) Young’s modulus
- \(E\) Plane strain Young’s modulus
- \(g(p, \lambda)\) Orthotropy function (Eqs. 1 and 4)
- \(G\) Energy release rate
- \(G_t\) Fracture resistance at unstable crack growth
- \(G_R\) Fracture resistance
- \(G_{ss}\) Steady-state toughness
- \(H\) Nondimensional function (Table I)
- \(S\) Loading span
- \(T\) Tearing modulus
- \(T_c\) Critical tearing modulus
- \(u_c\) Critical crack opening displacement
- \(W\) Specimen width
- \(Y(p)\) Orthotropy function (Eq. A-2)
- \(\Delta a\) Crack growth
- \(\Delta a_{ss}\) Crack growth at onset of steady state
- \(\lambda\) Elastic orthotropy parameter (Eq. 2)
- \(\mu\) Shear modulus
- \(\nu\) Poisson’s ratio
- \(\rho\) Elastic orthotropy parameter (Eq. 3)
- \(\sigma\) Stress
- \(\sigma_t\) Stress at onset of unstable crack growth
- \(\sigma_{ss}\) Stress at onset of crack growth
- \(\sigma_u\) Unnotched strength
- \(\sigma_n\) Characteristic bridging stress

**References**


