Effects of Matrix Cracks on the Thermal Diffusivity of a Fiber-Reinforced Ceramic Composite

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The present study focuses on the effects of a periodic array of matrix cracks on thermal diffusivity. The study is restricted to a simple unidirectional CFCC, loaded in tension parallel to the fiber axis. In situ measurements are made of both the longitudinal and the transverse diffusivities using a phase-sensitive photothermal technique. Ancillary measurements of crack density and crack opening displacement are used to determine the contributions to the longitudinal thermal resistance due to each crack. Additionally, models for thermal diffusivity of composite materials are used to elucidate the origins of the degradation in both the longitudinal and transverse diffusivities with applied stress. A complementary study on the effects of delamination cracks on the through-thickness thermal behavior is presented by McDonald et al.5

II. Experimental Techniques and Analysis

(1) Materials and Thermal Diffusivity Measurements

Measurements of thermal diffusivity were made using a phase-sensitive photothermal technique.4,5 The technique utilizes periodic heating at one point on the specimen surface and measurement of the phase lag ($\phi$) of the temperature with respect to the heat input at some other point. The heat source comprises a focused 0.5 W diode laser that is mounted on a precision x-y-z translation stage and modulated sinusoidally using a lock-in amplifier. Temperatures are measured at prescribed locations using fine Type T thermocouples. The thermocouples are affixed to the specimen surface by colloidal silver paint. The signal from the thermocouple is input into the lock-in amplifier, and, upon comparison with the input signal to the laser, the phase lag is obtained. To mitigate the effects of the thermal resistance of the thermocouple itself, the measurements are based on differences in phase lag relative to a reference measurement. For instance, the phase lag ($\phi_0(r)$) measured at the location nearest to the thermocouple, at a distance $r_0$, is then subtracted from all other measured values, thereby yielding phase-lag differences $\Delta \phi(\Delta r) = \phi(r) - \phi_0(r_0)$ where $\Delta r = r - r_0$. In this way, the phase lag associated with the thermocouple is removed and the resultant measurements become a function of only the thermal properties and the geometry of the test specimen. The diffusivities are then obtained by fitting the experimental data with the theoretical analysis presented in McDonald et al.5 and summarized below. When necessary, the corresponding thermal conductivities ($k$) are calculated in accordance with

$$k = \alpha \rho C_p$$

where $\alpha$ is the thermal diffusivity, $\rho$ the density and $C_p$ the specific heat at constant pressure.

Experimental measurements were made on a unidirectional composite comprising a magnesium aluminosilicate (MAS) matrix and 36 vol.% of ceramic grade Nicalon fibers. The longitudinal and transverse thermal diffusivities ($\alpha_L$ and $\alpha_T$) of the pristine material were determined from in-plane phase-lag measurements.
both parallel and perpendicular to the fiber direction. The through-thickness diffusivity and the specific heat were measured independently (by Holometrix Inc., Bedford, MA) using the laser flash technique.6

(2) Mechanical Tests and In Situ Diffusivity Measurements

The changes in thermal diffusivity because of matrix cracking were obtained from in situ thermal-phase lag measurements during tensile testing. The test specimen was 3 mm × 3 mm in cross-section and had a gauge length of 38 mm. One surface was polished to facilitate measurement of crack spacing via surface replicas. Fiberglass tabs were affixed to each specimen end with M-Bond adhesive. Strain gauges were mounted on each of the top and bottom surfaces. Two thermocouples were affixed to the top surface near the center and a third thermocouple was affixed to the bottom surface. The specimen ends were then inserted into wedge grips in a small screw-driven tensile fixture that was mounted directly below the laser. Prior to loading, a series of phase-lag measurements was made with each thermocouple. In the first of these measurements, the laser was positioned directly across from the thermocouple on the bottom surface and the phase lag was measured over the frequency range, \( f = 0.05 \text{ to } 2.0 \text{ Hz} \). In the next series, the laser frequency was fixed \( (f = 0.3 \text{ Hz}) \) and the phase lag was measured over a range of distances \( x = (0.25 \text{ to } 4.5 \text{ mm}) \) from each of the top thermocouple locations. Finally, the polished surface was replicated using cellulose acetate tape in a location close to one of the top thermocouples.

The specimen was then loaded in uniaxial tension parallel to the fiber direction. Once a significant number of matrix cracks had developed (with a spacing of \( \approx 150 \mu \text{m} \)), the test was interrupted and the phase-lag measurements were repeated. (At lower stresses and hence lower crack densities, the cracks had only a very small effect on diffusivity, as shown in Section III.) The polished surface was also replicated and the replicas were used to measure the crack spacing. Once one such set of measurements had been completed, the load was increased by a small amount, causing the formation of additional matrix cracks and further opening of the existing cracks, and the phase-lag measurements and surface replication were repeated again. Upon completion of several such measurements (up to loads slightly below that needed for fracture), the specimen was unloaded and the measurements repeated yet again.

(3) Analysis of Phase-Lag Measurements

The thermal diffusivities were obtained by fitting the phase-lag measurements with an analysis based on standard heat-flow theory. The analysis begins with the solution for the temperature along the top (heated) surface in an isotropic plate is defined by \( S \) to be infinite in length in the \( x \)-direction (perpendicular to the plane of the page).

\[
T = T_0 \exp\left(-\frac{\beta r (1 - i)}{r}\right) \quad (2)
\]

where \( i = (-1)^{1/2} \), \( \beta = (\pi \ell / \alpha)^{0.5} \), and \( T_0 \) a time-varying reference temperature. The phase lag of the temperature at \( r = 0 \) is defined by the argument of \( T \) in Eq. (2) and is given by

\[
\phi = r \beta = r (\pi \ell / \alpha)^{0.5} \quad (3)
\]

The effects of free surfaces on bodies of finite size can be incorporated through an image-source technique. For this purpose, an array of image sources is identified that yields the requisite boundary conditions on the free surfaces and the contributions to the temperature from all sources are added together. For plate specimens (such as those used to make measurements on the pristine materials), image sources are placed along a line perpendicular to the plane of the plate at intervals of \( \pm 2t \), satisfying adiabatic conditions at the top \((z = 0)\) and bottom \((z = t)\) surfaces of the plate (Fig. 1(a)). By analogy to Eq. (1), the phase lag of the temperature along the top (heated) surface in an isotropic plate is defined by the summed series

\[
S = |S| \exp(\phi) = \sum_{n=0, \pm 2, \pm 4, \ldots} \exp\left(-\frac{\beta r_n (1 - i)}{r_n}\right) \quad (4)
\]

where \( r_n \) is the distance from the \( n \)-th image source and the location of interest \( (r_n = (r^2 + n^2 \ell^2)^{1/2}) \).

For long rectangular bars, the image sources are placed on a rectangular array with separations of \( 2t \) and \( 2w \) in the directions parallel to the thickness and width directions, respectively (Fig. 1(b)). This array produces adiabatic conditions on the four surfaces of the bar. When the real heat source is located at the center of the surface of width \( 2w \), the phase lag along the center line of this surface in an isotropic bar is defined by

\[
S = |S| \exp(\phi) = \sum_{m=0, \pm 2, \pm 4, \ldots} \sum_{n=0, \pm 2, \pm 4, \ldots} \exp\left(-\frac{\beta r_{mn} (1 - i)}{r_{mn}}\right) \quad (5a)
\]

where \( r_{mn} \) is the distance from each image source to the point of interest \((x = \pm mw, z = \pm nt)\). Similarly, along the center line of
the bottom surface (opposite the surface being heated), the phase lag is defined by\(^5\)

\[
S = |S|\exp i\phi = \sum_{m = \pm 2, \pm 4, \ldots} \left[ \sum_{n = \pm 1, \pm 3, \ldots} \exp[-\beta r_m(1 - i)] \right]
\]

(5b)

To account for anisotropy, the preceding analysis is modified through a straightforward rescaling procedure. For orthotropic materials, the governing heat flow equation is transformed into that of an isotropic material through a rescaling of the principal axes, \(x, y, z\), in accordance with \(x' = x(\alpha_x)^{0.5}\), \(y' = y(\alpha_y)^{0.5}\) and \(z' = z(\alpha_z)\), where \(\alpha_x\), \(\alpha_y\) and \(\alpha_z\) are the diffusivities in the principal material directions and \(\alpha\) is chosen arbitrarily to be one of the three principal diffusivities.\(^7\)

(4) Crack Spacing Measurements

The average crack spacing, \(d\), at each load was determined from the surface replicas. The number of cracks intersecting a line of fixed length was counted in several regions on each replica in an optical microscope. The crack spacing was then calculated by dividing the sum of the line length by the total number of cracks counted.

(5) Crack-Opening Displacement Measurements

The average crack-opening displacement was determined from the strain and the crack spacing using a standard shear-lag model (see, for example, Aveston et al.\(^1\)). Prior to cracking, the longitudinal strain in the matrix and the fibers are equal to one another. Upon cracking, the matrix contracts longitudinally in the region close to the crack (where debonding has occurred) whereas the fibers extend because of the load transferred from the matrix. The total crack-opening displacement is the sum of these two contributions: \(\delta = \Delta u_t + \Delta u_m\) where \(\Delta u_t\) is the fiber extension and \(\Delta u_m\) the matrix contraction. Assuming that the length of the slipped regions from adjacent cracks do not overlap, the fiber extension is \(\Delta u_t = \varepsilon_m \cdot d\) where \(\varepsilon_m\) is the inelastic strain caused by matrix cracking. This strain is defined by \(\varepsilon_m = \varepsilon - \sigma E\) where \(\sigma\) is the applied tensile stress and \(E\) the longitudinal composite modulus. Along the crack surface, the load is carried solely by the fibers; here the matrix strain is zero and the fiber strain is \(\sigma v_f E_f\), where \(E_f\) is the Young’s modulus of the fibers and \(v_f\) the fiber volume fraction. Over the slip length, load is transferred back to the matrix from the fibers through interfacial friction such that equal strains, \(\sigma E\), are obtained in the two phases at the point where the fibers and the matrix are bonded. This yields a matrix contraction of

\[
\Delta u_m = \frac{\Delta u_t (1 - v_f) E_m}{v_f E_f} = \frac{\varepsilon_m (1 - v_f) E_m}{v_f E_f}
\]

where \(E_m\) is the Young’s modulus of the matrix. The resulting crack opening displacement is

\[
\delta = \Delta u_t + \Delta u_m = \frac{\varepsilon_m E}{(1 - v_f) E_m}
\]

III. Experimental Results

Typical measurements and the corresponding curve fits for the diffusivities of the pristine material are presented in Fig. 2. The average values obtained from several sets of measurements are \(\alpha_x' = 1.35 \times 10^{-6} \text{ m}^2/\text{s}\) and \(\alpha_y' = 1.08 \times 10^{-6} \text{ m}^2/\text{s}\). The latter value is essentially the same as the one measured via the laser-flash method in the through-thickness direction on the same material \((1.04 \times 10^{-6} \text{ m}^2/\text{s})\), as expected from the transversely-isotropic symmetry of the composite.

The results pertaining to the stress-strain response, the crack spacings and the crack-opening displacements are summarized in Figs. 3–5. The solid symbols on the stress-strain curves denote the points at which the test was interrupted and the diffusivity measurements were made. The cracks developed progressively over the range of stresses of interest, from \(\approx 200\) MPa to 400 MPa. Additionally, the average crack-opening displacement increased quadratically with stress. The latter trend is consistent with the prediction from a shear-lag model, assuming that the interface-sliding stress is constant along the debonded portion.
of the interface. The solid line shown on Fig. 4 is a fit from such a model using a sliding stress $\tau = 30$ MPa. Upon unloading, the crack-opening displacement recovered to about 10% of its peak value (from $\sim 0.3$ μm to 0.03 μm). An additional notable feature is the somewhat discontinuous nature of the matrix cracks (seen in Figs. 5(b) and (c)).

The variations in the thermal diffusivities with stress are plotted in Fig. 6. The longitudinal diffusivity, $\alpha_L$, decreased approximately linearly with stress beyond the matrix-cracking limit to a level of $\sim 80\%$ of its initial (pristine) value at the peak stress; upon unloading, it recovered to within $\sim 3\%$ of its initial value. This recovery is attributable to the closure of the matrix cracks (Fig. 4). Similar reductions were obtained in the transverse diffusivity with increasing stress. The latter reduction is attributable to the debonding and sliding that occur along the fiber/matrix interface adjacent to the matrix cracks. In contrast to the longitudinal diffusivity, the transverse diffusivity did not recover appreciably upon unloading.

![ Optical micrographs of surface replicas showing the development of cracks at one particular region on the polished surface, at stress levels of (a) 0, (b) 275 MPa, and (c) 380 MPa. ]

**Fig. 5.** Optical micrographs of surface replicas showing the development of cracks at one particular region on the polished surface, at stress levels of (a) 0, (b) 275 MPa, and (c) 380 MPa.

**IV. Analysis**

(1) **Pristine Material**

The thermal diffusivity of the pristine composite can be rationalized in terms of models for heat flow through multiphase media, taking into account the topology and the thermal properties of the phases and the conductance of the interface between the phases. For steady state heat flow, the longitudinal diffusivity is obtained from a simple parallel resistance model; the result is

$$\alpha_L = \frac{k_L}{\rho C_p} = \frac{\nu L (1 - \nu) k_m}{\rho C_p}$$

where $k_L$ is the longitudinal composite conductivity, and $k_m$ is the thermal conductivities of the fiber and the matrix, respectively. (This diffusivity is independent of the interface conductance because the heat flux across the interface is zero.) Upon combining the measured values of $\alpha_L$, $\rho$, $C_p$ and $\nu$ with the reported value of $k_L$ (1.8 W/(mK))$^9$ and Eq. (8), the matrix conductivity is inferred to be $k_m = 2.9$ W/(mK)$^{-1}$ (Table I).

The transverse conductivity ($k_T$) under steady-state heat flow has been obtained using an effective medium approach.$^{10,11}$ Combining this result with Eq. (1) yields the transverse diffusivity

$$\alpha_T = \frac{k_T}{\rho C_p} = \frac{k_m}{\rho C_p} \left[ \frac{1 + \frac{k_f}{k_m} + \frac{1}{B_i} v_f}{\frac{1}{k_m} + \frac{1}{B_i} v_f} \right]$$

where $B_i$ is the interface Biot number, defined by $B_i = R h_i/k_L$ with $R$ is the fiber radius (7.5 μm) and $h_i$ is the interface conductance. Using Eq. (9) with the known values of the various constituent and composite properties (Table I), the interface Biot number is inferred to be $B_i = 1.1$. The results in Eqs. (8) and (9) also apply to transient and periodic heat flow conditions provided that the specimen size is significantly larger than the size scale of the microstructure, as characterized by the fiber spacing.$^{12}$ In the present experiments, the ratio of size scales is $>100$ and hence the preceding results apply.

(2) **Cracked Material**

The effective medium model can also be used to rationalize the transverse diffusivity of the composite following matrix cracking. In this case, the cracks are oriented parallel to the direction of heat flow and therefore have no direct effect on diffusivity. However, the debonding and sliding that accompany matrix cracking alter the interface conductance. Upon applying Eq. (9) to the transverse-diffusivity data in Fig. 6, the average interface Biot number is calculated and is found to...
decrease with increasing stress from an initial value \( \approx 1 \) to \( \approx 0.3 \) at the peak stress (Fig. 7). Furthermore, the value of \( B_i \) remains essentially unchanged after unloading from the peak stress; the inference is that the reduction in the interface conductance due to debonding and sliding is permanent. The lack of recovery in \( B_i \) after unloading coupled with the large recovery in the crack-opening displacement (from \( \approx 0.3 \) to 0.03 \( \mu \)m) further supports the assertion that the cracks themselves do not have a direct effect on the transverse diffusivity.

The effects of the cracks on the longitudinal diffusivity can be described by a model developed by Lu and Hutchinson. The model is based on a heat-flow analysis of a unit cell comprising concentric cylinders of fiber and matrix and a matrix crack at the midpoint of the cell (Figs. 8(a) and (b)). A tacit assumption in this cell model is that each matrix crack spans the entire specimen width; this idealization differs slightly from the actual crack morphology in the present composite and has implications in the accuracy of the model predictions, as described later. Approximate analytical solutions for the longitudinal diffusivity of this cell have been obtained, following an approach analogous to shear lag for the stiffness loss of a cracked composite. The model has been validated through finite element calculations and through a more rigorous analytical solution.

Assuming that the interface has uniform conductance along the entire length, \( k_L \) is predicted to be:

\[
\frac{k_L^c}{k_L} = 1 + \left[ \frac{1}{1 - \nu_i k_m} \left( \frac{\tanh \frac{sd}{2R}}{1 + \left( 2k_m^c/\nu_i k_m \right) \tanh \frac{sd}{2R}} \right) \right]^{-1/2}
\]

where \( B_i \) is Biot number for the matrix crack defined by \( B_i = H_i R / k_m \), with \( H_i \) the conductance of the matrix crack, and \( \nu \) a dimensionless parameter, defined by:

\[
\nu = \left( \frac{8k_m^c}{1 - \nu_i k_m} \right)^{1/2}
\]

For the entire range of parameter values pertinent to the present study, the argument of the tanh term in Eq. (10) is \( \gg 1 \), and consequently, the tanh term is taken to be unity. (This approximation yields an error <0.01% when the argument is \( \approx 5 \)). With this approximation, the predicted conductivity reduces to:

\[
\frac{k_L^c}{k_L} = 1 + \frac{2R(1 - \nu_i)k_m}{dK(\nu_i + 2\nu_i^2 R B_i k_m)}
\]

### Table I. Summary of Constituent and Composite Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_f )</td>
<td>1.8 W/(m·K)(^{-1})</td>
<td>Reference 8</td>
</tr>
<tr>
<td>( k_m )</td>
<td>2.9 W/(m·K)(^{-1})</td>
<td>Reference 1</td>
</tr>
<tr>
<td>( E_f )</td>
<td>200 GPa</td>
<td></td>
</tr>
<tr>
<td>( E_m )</td>
<td>115 GPa</td>
<td></td>
</tr>
<tr>
<td>( 2R )</td>
<td>15 ( \mu )m</td>
<td>Measured(^{1})</td>
</tr>
<tr>
<td>( \alpha_{L}^{0} )</td>
<td>1.35 \times 10^{-6} m²/s</td>
<td>Measured(^{1})</td>
</tr>
<tr>
<td>( \alpha_{T}^{0} )</td>
<td>1.08 \times 10^{-6} m²/s</td>
<td>Measured(^{1})</td>
</tr>
<tr>
<td>( \rho )</td>
<td>2.61 g/cm³</td>
<td>Measured(^{1})</td>
</tr>
<tr>
<td>( C_p )</td>
<td>0.704 J/(g·K)</td>
<td>Measured(^{1})</td>
</tr>
<tr>
<td>( k_f^0 )</td>
<td>2.48 W/(m·K)(^{-1})</td>
<td>( \rho C_p \alpha_{L}^{0} )</td>
</tr>
<tr>
<td>( k_m^0 )</td>
<td>1.99 W/(m·K)(^{-1})</td>
<td>( \rho C_p \alpha_{L}^{0} )</td>
</tr>
<tr>
<td>( E )</td>
<td>145 GPa</td>
<td>Measured(^{1})</td>
</tr>
<tr>
<td>( \nu_i )</td>
<td>0.36</td>
<td>Measured(^{1})</td>
</tr>
</tbody>
</table>

\(^{1}\)This study. \(^{1}\)Holometrix, Inc., Bedford, MA.

At ambient temperature, \( H_i \) is dictated by conduction through the air gap between the two faces of the matrix crack. The connection between the crack conductance and the crack-opening displacement is dictated by the magnitude of the Knudsen number, defined by \( Kn = \lambda / d \) where \( \lambda \) is the mean free path of the air molecules within the gap. According to the kinetic theory of gases, \( \lambda \) is given by \( \lambda (m) = 2.27 \times 10^{-5} T / P \) where \( T \) is the absolute temperature and \( P \) is the pressure (Pa). At ambient temperature and pressure, \( \lambda \approx 0.066 \mu \)m. When \( Kn << 1 \), the temperature of the air on either side of the crack is essentially equivalent to the temperature of the respective crack face and the temperature distribution across the gap is linear. In this regime, the crack conductance is given by:

\[
H_i = \frac{k_s}{\delta}
\]

where \( k_s \) is the thermal conductivity of air (\( \approx 0.026 \) W/(m·K)\(^{-1}\)) at ambient temperature. Conversely, when \( Kn \) is a significant fraction of unity (or larger), the temperature of the air adjacent to the crack face is different from that of the crack face itself and the profile across the gap is nonlinear. This is a consequence of the finite distance over which atomic collisions must occur in order to communicate the temperature differences across the gap. In this regime, the crack conductance can be expressed approximately by (see Appendix)

\[
H_i = \frac{k_s}{\delta(1 + 4Kn)}
\]

![Fig. 8. (a) and (b) Schematics of the unit cell of the cracked composite; (c) series resistor representation of the longitudinal-thermal resistance.](image)
In the limit where $Kn \to 0$, the result in Eq. (14) reduces to that in Eq. (13), as required. Furthermore, when $Kn >> 1$, the conductance is given by $H_\perp \approx k f/4 L$, independent of $\delta$.

For the subsequent analysis, it is convenient to re-interpret the result in Eq. (12) in terms of the series-resistor model depicted in Fig. 8(c). One of the resistors in Fig. 8(c) represents the longitudinal thermal resistance per unit length of the pristine material, $1/k_0$. The other resistor represents all of the additional thermal resistance due to the matrix crack and the constriction resistance of the bridging fiber; it is defined by $1/dH$ where $H$ is the effective crack conductance. With this model, the predicted longitudinal conductivity is

$$\frac{k_i}{k_0} = 1 + \frac{v_i}{k f} \left( 1 - \frac{v_i}{k f} \right) B_i + \frac{v_i}{k f} \left( 1 - \frac{v_i}{k f} \right) H_\perp \left( 1 + \frac{4}{B_i} \right)^{1/2}$$

(15)

where $B_i$ is the effective Biot number of the bridged crack, defined by $B_i = H R_0/k_0$. This is a key result in the sense that it reveals the combined effects of the normalized crack density and the crack Biot number on the reduction in the composite conductivity.

Upon comparing Eqs. (12) and (15), the effective crack conductance from the model of Lu and Hutchinson becomes

$$\bar{H} = \frac{s v_i k f k_0}{2(1 - v_i) k m R} + \frac{k_i}{k_0} \left( \frac{H_\perp}{1 - v_i} \right)^2$$

(16)

The first term on the right side of Eq. (16) is the contribution from the bridging fiber and is independent of $\delta$; the second term is the contribution from the air within the matrix crack. This result can be expressed alternatively in terms of the corresponding Biot number (by combining Eqs. 11 and 16):

$$\bar{B} = \frac{k_i}{k_0} \left( 1 + \frac{v_i}{k f} \right) B_i + \frac{v_i}{k f} \left( 1 - \frac{v_i}{k f} \right) H_\perp \left( 1 + \frac{4}{B_i} \right)^{1/2}$$

(17)

For the purpose of comparing the model predictions with the experimental measurements, the effective conductance of each crack was determined from the measured conductivities through Eq. (15). The results are plotted against $\delta$ in Fig. 9. Then, the predicted conductances (from Eqs. 14 and 16) were calculated using $B_i$ as an adjustable parameter, over the range $0.1-100$. Comparisons between the two are shown in Fig. 9. Also shown are the limiting values of the predicted crack conductance as $Kn \to \infty$ (shown by (- - -) on the left) and as $H_\perp \to 0$ (shown by (--- - - -) on the right. Using Biot numbers that are consistent with the transverse diffusivity measurements, the predicted conductances are somewhat lower than the measured values. For instance, at the larger values of $\delta(\approx 0.25 \mu m)$ where $B_i \approx 0.3$, the predicted conductance $(1.1 \times 10^6 \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1})$ is $\approx 30\%$ below the measured value $(1.6 \times 10^6 \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1})$. At lower values of $\delta(\approx 0.15 \mu m)$ where $B_i \approx 1$, the predicted value $(1.6 \times 10^6 \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1})$ is about half of the measured one $(3 \times 10^6 \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1})$. It is surmised that these differences are attributable to the fact that the matrix cracks do not span across the entire specimen cross-section and hence there remain contiguous matrix segments parallel to the fiber axis. These segments enhance $\alpha_f$ and hence increase the inferred crack conductance. Despite the discrepancies in the absolute values of $\bar{H}$, it is notable that the rate at which $\bar{H}$ decreases with $\delta$ is greater than that predicted for a constant value of $B_i$. This trend is consistent with the notion that the interface conductance degrades with increasing crack opening, as inferred from $\alpha_f$ (Fig. 7). The large recovery in the crack conductance after unloading may be attributed to crack closure effects. Crack closure would have the effect of bringing the matrix crack faces in direct contact with one another at discrete points and further enhance the crack conductance.

The model of Lu and Hutchinson can be used further to draw insights into the relative importance of $B_i$ and $B_i$ in $\bar{B}$ (the latter being the key parameter dictating the reduction in composite conductivity via Eq. (15)). To illustrate some of the important trends, Fig. 10 presents a map with the combinations of $B_i$ for which $\bar{B} = 1$ in composites with $v_i = 0.4$ (from Eq. (17)). Each of the lines is for a prescribed conductivity ratio, $k_f/k_m$. Three limiting cases emerge. (i) When $B_i \ll 1$, a critical value of $B_i$ is required to attain the prescribed value of $\bar{B}$, defined by

$$B_i = \frac{k_i}{k_0} \left( 1 + \frac{v_i}{k f} \right)$$

(18)

These limiting values are plotted as the horizontal dotted lines on the left side of Fig. 10. (ii) Conversely, when $B_i \ll 1$, a critical value of $B_i$ is required, defined by

$$B_i = \frac{2}{\bar{B}^2} \left( \frac{v_i}{k f} \right)^4 \frac{1}{1 + \frac{v_i}{k f} \left( 1 + \frac{4}{B_i} \right)^{1/2} - 1}$$

(19)

Strictly, this result only applies when $\delta \ll R$. Otherwise, the thermal resistance of the portion of the fiber contained within the crack must be taken into account.
This result applies only when the denominator is positive, i.e.,

\[
    2 \left( \frac{v_i k_t}{1 - v_i k_m} \right)^2 \left( 1 + \frac{v_i k_t}{1 - v_i k_m} \right) > 1
    \tag{20}
\]

The limiting values of \( B_i \) are plotted as the dotted vertical lines for the values of \( k_i/k_m \), for which Eq. (19) is satisfied. (iii) When Eq. (20) is not satisfied (at low values of \( k_i/k_m \), for instance), a critical value of \( B_i \) is required, even as \( B_i \to \infty \). In order to attain the prescribed value of \( B \). This critical value is given by

\[
    B_i = \frac{k_i \left( 1 + \frac{v_i k_t}{1 - v_i k_m} \right)^{1/2}}{1 - v_i k_m} \left( 1 + \frac{v_i k_t}{1 - v_i k_m} \right)^{1/2}
    \tag{21}
\]

These values are plotted as dashed horizontal lines in the right side of Fig. 10 for the appropriate values of \( k_i/k_m \). It is expected that such maps, used in conjunction with the crack density, would be useful in identifying the combinations of thermal properties for which the longitudinal composite conductivity remains at acceptable levels (through Eqs. (15) and (17)).

V. Summary

Changes in thermal diffusivity due to matrix cracking and interface debonding and sliding in a unidirectional CFCC have been investigated. The key results include: (i) a progressive reduction in the longitudinal diffusivity with increasing stress, resulting from the presence of matrix cracks and the constringency resistance of bridging fibers; (ii) a comparable reduction in the transverse diffusivity with increasing stress, resulting from a degradation in the interface conductance associated with debonding and sliding; (iii) almost complete recovery in the longitudinal diffusivity after unloading; and (iv) essentially no recovery in the transverse diffusivity after unloading. The recovery of the longitudinal diffusivity highlights the important role of stress in the thermal diffusivity. Indeed, had the longitudinal diffusivity measurements been made only in the unloaded state (after tensile testing), the effects of the cracks would have been overlooked altogether. Furthermore, the lack of recovery in the transverse diffusivity attests to the irreversibility of the degradation in the interface conductance following debonding and sliding. The effects of finite crack lengths and crack closure require further investigation.

Appendix

Matrix Crack Thermal Conductance

When the mean free path, \( \lambda \), of the air molecules is a significant fraction of the crack opening displacement, the temperature of the solid at the crack surface is different from the air temperature at the same location and the temperature distribution of the air near the crack surfaces is highly nonlinear over distances that are proportional to \( \lambda \). Heat conduction across the gap can be analyzed in an approximate way by extrapolating the nearly-linear portion of the temperature distribution near the center of the gap to the crack faces. Upon performing such an extrapolation, the temperature jump \( \Delta T \) that is obtained at each of the two crack faces is given by

\[
    \Delta T = \left( T_1 - T_2 \right) \left( \frac{\Omega}{1 + 2\Omega} \right)
    \tag{A-1}
\]

where:

\[
    \Omega = \left( \frac{2 - \eta}{\eta} \right) \left( \frac{9\gamma - 5}{2(\gamma + 1)} \right) k_n
    \tag{A-2}
\]

Here \( \eta \) is the thermal accommodation coefficient (typically close to unity for ambient air); \( \gamma \) is the ratio of the constant-pressure specific heat to the constant-volume specific heat of air; and \( T_1 \) and \( T_2 \) are the temperatures of the two crack faces. The heat flux per unit area across the air gap is:

\[
    q = \left( \frac{k_n}{\delta} \right) (T_1 - T_2 - 2\Delta T)
    \tag{A-3}
\]

where the term \( T_1 - T_2 - 2\Delta T \) represents the effective temperature change across the air gap. Combining the results in Eqs. (A-1)-(A-3) yields the effective crack conductance:

\[
    H_e = \frac{q}{T_1 - T_2} = \frac{k_n}{\delta(1 + 2\Omega)}
    \tag{A-4}
\]

Substituting the values \( \eta = 0.9 \) and \( \gamma = 1.4 \) into Eq. (A-2) yields \( \Omega \approx 2Kn \) and Eq. (A-4) then reduces to Eq. (14).

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References